

necessarily a good assumption, particularly for rocket motor conditions.⁴ A more general expression for R_p , including radiative heat feedback, is [see Eq. (20) in Ref. 5, Eq. (7) in Ref. 2, Eq. (37) in Ref. 6, or Eq. (31) in Ref. 4]

$$R_p = \frac{\nu + \delta(\lambda - 1)}{\lambda r + \frac{k}{\lambda} - (r + k) + 1 - \frac{k\tilde{Q}_r(\lambda - 1)}{\lambda(\beta + \lambda - 1)}} \\ = \frac{nAB + n_s(\lambda - 1)}{\lambda + \frac{A}{\lambda} - (1 + A) + AB - \frac{A\tilde{Q}_r(\lambda - 1)}{\lambda(\beta + \lambda - 1)}} \quad (13)$$

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Reply by the Author to M. Q. Brewster and S. F. Son

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THE paper under discussion¹ was devoted to the problem of intrinsic burning stability of solid energetic materials. The analysis was performed by 1) a general nonlinear approach for chemically inert condensed-phase and 2) a less general linear frequency response function to include condensed-phase reactions (for which the general approach could not be applied); both approaches were developed in the flame modeling framework. The results obtained by the authors of the Comment² in the Zeldovich–Novozhilov (ZN) framework were mentioned. However, the objective was understanding of

intrinsic burning stability, not of frequency response function (although this can help as indeed shown in the paper). The Comment² only pays attention to details of the ZN linear frequency response function for chemically inert condensed phase.

Within this framework, the following is our Reply.

The general Arrhenius pyrolysis function was tacitly assumed of the form

$$r_{b, \text{Arr}} = \tilde{A}_s p^{n_s} T_s^{n_{sq}} \exp[-(\tilde{E}_s / R T_s)] \quad (1)$$

with $n_{sq} = 0$ for lack of experimental data.

Equation (46) (Ref. 1) should read $n_{sq} = \delta_q / r$.

It is true that Eq. (42) (Ref. 1), although general, is limited to $n_{sq} = 0$.

Both Arrhenius and KTSS pyrolysis laws are fine if the proper compatibility relationships are used. For pressure-driven burning only, these compatibility relationships (related to the pyrolysis Jacobian) are discussed in detail for Arrhenius pyrolysis in Ref. 3 and for KTSS pyrolysis in Ref. 4. For pressure-driven burning with arbitrary initial temperature, the proper compatibility relationships are discussed in Ref. 5. A more general treatment allowing for radiation-driven burning is under preparation. Notice that in the paper under discussion, initial temperature is not a parameter (see line 3 subsection II.A on page 806, Ref. 1). In addition, please consider that any mathematically smooth and increasing (with temperature) pyrolysis laws are allowed for the general nonlinear approach developed to test intrinsic burning stability. The Arrhenius surface pyrolysis is a particular pyrolysis law used to compute linear frequency response functions only, whereas the KTSS surface pyrolysis was never implemented.

The overall problem of simultaneous pressure- and radiation-driven burning is fully analyzed under broad terms in of Ref. 1, Fig. 4 page 810; an even more general treatment, allowing for radiation penetration, is in Ref. 6. Whether to use B_p or B_q is irrelevant, as shown by Eqs. (53) and (54) on page 810 of Ref. 1, whereby several parameters used in the literature, including B_q or B_p , are simply particular expressions of the more general a and b formalism (see definitions on page 807 of Ref. 1). Perhaps the confusion is because of another minor, but important, misprint: Ref. 1, Eq. (54) should read

$$K = q_{DB} = A_{KTSS} = 1 + A - AB_q = b = 1 + k/r - 1/r \quad (2)$$

(b disappeared in the final print).

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